

Searchable Abstracts Document

SIAM Conference on Discrete Mathematics (DM24)

July 8–11, 2024 Spokane Convention Center, Spokane, Washington, U.S.

This document was current as of June 17, 2024. Abstracts appear as submitted.



3600 Market Street, 6th Floor Philadelphia, PA 19104-2688 U.S. Telephone: 800-447-7426 (U.S. & Canada) +1-215-382-9800 (Worldwide) meetings@siam.org

IP1

A New Lower Bound for Sphere Packing

What is the maximum proportion of d-dimensional space that can be covered by non-overlapping, identical spheres? In this talk I will discuss a new lower bound for this problem, which is the first asymptotically growing improvement to Rogers' bound from 1947. In contrast to most previous work, our proof is almost entirely combinatorial and reduces to a novel theorem about independent sets in graphs with bounded degrees and codegrees. This is based on joint work with Marcelo Campos, Matthew Jenssen and Marcus Michelen.

<u>Julian Sahasrabudhe</u> University of Cambridge jdrs2@cam.ac.uk

IP2

Computational Complexity in Algebraic Combinatorics

Algebraic Combinatorics studies objects and quantities originating in Algebra, Representation Theory and Algebraic Geometry via combinatorial methods, finding formulas and neat interpretations. Some of its feats include the hook-length formula for the dimension of an irreducible symmetric group (S_n) module, or the Littlewood-Richardson rule to determine multiplicities of GL irreducibles in tensor products. Yet some natural multiplicities elude us, among them the fundamental Kronecker coefficients for the decomposition of tensor products of S_n irreducibles, and the plethysm coefficients for compositions of GL modules. Answering those questions could help Geometric Complexity Theory towards establishing lower bounds for the far-reaching goal to show that $P \neq NP$. We will discuss how Computational Complexity Theory provides a theoretical framework for understanding what kind of formulas or rules we could have. As a proof of concept we show that the square of a symmetric group character could not have a combinatorial interpretation. Based on joint works with Christian Ikenmeyer and Igor Pak.

<u>Greta Panova</u> University of Southern California gpanova@usc.edu

IP3

Random Cayley Graphs and Combinatorial Perspectives on Additive Number Theory Problems

Cayley graphs provide interesting bridges between graph theory, additive combinatorics and group theory. Fixing an ambient finite group, random Cayley graphs are constructed by choosing a generating set at random. These graphs reflect interesting symmetries and properties of the group, at the cost of inducing complex dependencies. I will discuss several results on clique and independence numbers of random Cayley graphs in general groups, progress towards a conjecture of Alon on existence of Ramsey Cayley graphs, and a proof of a conjecture of Alon and Orlitsky. These questions are naturally connected with some fundamental problems in additive number theory. Surprisingly, our insights suggest that in many of these problems, the group structure is superfluous and can be replaced by much more general combinatorial structures. Taking these insights further, I will discuss several combinatorial generalizations of important structural results in additive number theory, as well as new corollaries in additive number theory. Based on joint work with David Conlon, Jacob Fox and Liana Yepremyan.

Huy T. Pham Stanford University huypham@stanford.edu

IP4

Complex Discrete Probability Models in Evolutionary Biology: Challenges and Opportunities

The reconstruction of species phylogenies from genomic data is a key step in modern evolutionary studies. This task is complicated by the fact that genes evolve under biological phenomena that produce discordant histories. These include hybrid speciation, horizontal gene transfer, gene duplication and loss, and incomplete lineage sorting, all of which can be modeled using random gene tree distributions building on well-studied discrete stochastic processes (branching processes, the coalescent, random rearrangements, etc.). Gene trees are in turn estimated from molecular sequences using Markov models on trees. The rigorous analysis of the resulting complex models can help guide the design of new reconstruction methods with computational and statistical guarantees. I will illustrate the challenges and opportunities in this area via a few recent results. No biology background will be assumed.

<u>Sebastien Roch</u> University of Wisconsin, Madison roch@math.wisc.edu

IP5

Induced Subgraphs and Treewidth

Treewidth is a measure of the complexity of a graph and has both structural and algorithmic consequences. While results of Robertson and Seymour characterize which minors appear in graphs of large treewidth, the same question is still open for induced subgraphs. I will present some recent results towards an answer to this question, and about the related notion of tree independence number.

Sophie Spirkl University of Waterloo sophie.spirkl@uwaterloo.ca

IP6

Random Spanning Trees and Applications to the Traveling Salesperson Problem

I will give a quick introduction to the field of geometry of polynomials: namely to encode a discrete phenomenon in coefficients of complex multivariate polynomials and studying the underlying combinatorial structure via the interplay of the coefficients, zeros, and function values of these polynomials. I will use this theory to explain several new properties of random spanning tree distributions. Finally, I will present applications in designing approximation algorithms for the traveling salesperson problem and the thin tree conjecture.

Shayan Oveis Gharan University of Washington shayan@cs.washington.edu

IP7

Presentation and Closing Remarks: A Constant

Lower Bound for Frankl's Union-closed Sets Conjecture

A finite set system is union-closed if for every pair of sets in the system their union is also in the system. Frankl in 1979 conjectured that for any such system there exists an element which is contained in of the sets in that system (the only exception being the family containing just the empty set). In this talk I will discuss how a simple observation regarding the contrapositive of Frankl's conjecture eventually led to the discovery of an information theoretic approach on the problem and a proof of the first constant lower bound.

<u>Justin Gilmer</u> Google DeepMind gilmer@google.com

$\mathbf{SP1}$

2024 Dnes Knig Prize Lecture: When Are Structures Robust Under Randomness?

Given a collection of substructures of a large discrete system, when are they robust under random subsampling of the system? For instance, among the substructures in the collection, we may require at least one complete substructure to survive the subsampling; or alternatively, we may ask for "most" of a substructure, for appropriate notions of "most", to survive. This theme entails a number of important questions in probabilistic combinatorics; including in particular questions about thresholds in random graphs and hypergraphs. The Kahn-Kalai conjecture predicts that a natural necessary condition to guarantee a complete substructure to survive the random subsampling is also roughly sufficient. Interestingly, this condition on the collection of substructures so-called "being not psmall" also appears in other contexts, and relates to other notions of robustness. I will discuss recent developments around robustness of structures under randomness, as well as new understanding on how to verify conditions for robustness in applications of interest.

Huy T. Pham Stanford University huypham@stanford.edu

Jinyoung Park New York University jinyoungpark@nyu.edu

JP1

Joint Plenary Speaker with the 2024 SIAM Annual Meeting (AN24): New Frontiers in Structure vs Randomness with Applications to Combinatorics, Complexity, Algorithms

In 1936, Erdos and Turan asked the following: Suppose you have a set S of integers from $1, 2, \ldots, N$ that contains at least N/C elements. Then, for large enough N, must S have three equally spaced numbers (i.e., a 3-term arithmetic progression)? In 1946, Behrend showed that C can be at most $exp(\sqrt{(\log N)})$. Since then, the problem has been a cornerstone of the area of additive combinatorics, with the best bound being $C = (\log N)^{(1+c)}$ for some constant c > 0. Recent work obtained an exponential improvement showing that C can be as big as $exp((\log N)^{0.09})$, thus getting closer to Behrend's construction. In this talk, I will describe this result and the main ingredient, a new variant of the "structure vs. randomness" paradigm. The latter is an

party protocols. 2. Algorithm design: fast combinatorial algorithms for Boolean matrix multiplication, detecting triangles in graphs. Based on works with Amir Abboud, Nick Fischer, Zander Kelley, Shachar Lovett.

Raghu Meka University of California, Los Angeles raghum@cs.ucla.edu

CP1

A Graph Partitioning Problem

A connected graph, G, of order n is said to be arbitrarily partitionable if, for every integer partition (a_1, \ldots, a_k) of n, there exists an (a_1, \ldots, a_k) -partition of V so that each subgraph induced on the selected a_i vertices is connected. The study of arbitrarily partitionable graphs is motivated by a resource allocation problem in computer science where k users sharing a network are each competing for a_i resources. In this context, the subgraph induced by the set of resources attributed to each user should be connected and each resource should only be allocated to one user. In this presentation, we consider a recursive version of arbitrarily partitionable graphs in which the network resources must be distributed as evenly as possible to exactly two users. Originally conceived by Dean Hoffman, we will present preliminary results towards characterizing those graphs which have such a distribution, a connection to evolutionary biology, and a discussion of the complexity of the problem.

<u>Ann W. Clifton</u>, Galen Turner Louisiana Tech University aclifton@latech.edu, gturner@coes.latech.edu

Hays Whitlatch Gonzaga University whitlatch@gonzaga.edu

CP1

Weighted Enumeration of Nonbacktracking Walks on Weighted Graphs

Identifying important nodes in a network is a key feature of complex network analysis [Paolo Boldi and Sebastiano Vigna, Axioms for centrality, 2014]. This is done using centrality measures: functions that assign a non-negative real value to each node in the network, thereby inducing a ranking of the nodes reflective of their relative importance in the network. Backtracking walks contain a sequence of nodes of the form uvu which can be unrealistic within the context of a network model. One example of such a context is an instant messaging network, in which it is unlikely that one reports received information back to its messenger. Furthermore, discounting such walks is known to offer concrete benefits such as avoiding localisation in the eigenvectors of the non-backtracking adjacency matrix [Tatsuro Kawamoto, Localized eigenvectors of the non-backtracking matrix, 2016]. For weighted and temporal networks, the enumeration of the correctly weighted number of non-backtracking walks has been complicated by the possibility of non-uniform edge weights or the possibility for walks to backtrack not only spatially, but also temporally. The subject of this talk will be the multigraph approach which provides a combinatorially correct formula for non-backtracking centrality measures defined by analytic functions for weighted temporal networks [Francesca Arrigo, Desmond J. Higham, Vanni Noferini and Ryan Wood, Weighted enumeration of nonbacktracking walks on weighted graphs, 2024]

Ryan Wood

Aalto University ryan.wood@aalto.fi

$\mathbf{CP2}$

The Generalized Honeymoon Oberwolfach Problem

The Honeymoon Oberwolfach Problem (HOP), introduced by ajna, is one of the most recent variants of the classic Oberwolfach Problem. This problem asks whether it is possible to seat $2m_1 + 2m_2 + \ldots + 2m_t = 2n$ participants consisting of n newlywed couples at t round tables of sizes $2m_1, 2m_2, \ldots, 2m_t$ for 2n-2 successive nights so that each participant sits next to their spouse every time and next to each other participant exactly once. HOP has been studied by Jerade, Lepine and ajna, and some significant cases of it have been solved. Until now, HOP has been restricted by the condition that each table must seat at least four people. In this talk, I generalize the problem to allow for tables of size two. In the generalized HOP, we are aiming to seat the 2n participants at s tables of size 2 and t round tables of sizes $2m_1, 2m_2, \ldots, 2m_t$ with the assumption that $2n = 2s + 2m_1 + 2m_2 + \ldots + 2m_t$. I will present a general approach to this problem, as well as recent solutions to several cases.

<u>Masoomeh Akbari</u>, Mateja Šajna University of Ottawa makba074@uottawa.ca, msajna@uottawa.ca

$\mathbf{CP2}$

Cover-free Families on Hypergraphs

Using cover-free families (CFFs) in group testing for finding defective items were studied by Hwang and Ss in 1986. We use CFFs to construct the pool of items assuming that a small number d of infected items are in a pool of n items. Consider a family F = (X, B) where X is a set of t points and B is a set of n subsets of X, and for $d < t \leq n$, such a family is called *d*-CFF (t, n), if for any d+1 subsets B_0, B_1, \cdots, B_d , no set is contained in the union of d others. We denote by t(d, n) the minimum t for which there exists a d-CFF(t, n). The optimal 1-CFF can be achieved due to Sperner's theorem. For $d \geq 2$, we rely on the asymptotic bounds of t(d, n). Some optimal/best-known values of t(2, n) for small values of n are given by Li, van Rees and Wei(2006). In this talk, we extend the definition of a CFF to include a hypergraph H. If we associate the vertices with the items to be tested and hyperedges with items in interactions with each other, this generalized CFF, \overline{H} -CFF, would identify them as long as they are connected by a hyperedge. The goal is to improve the efficiency and utilize the internal properties to optimize the CFFs. In case of graphs, if some pairs of vertices do not interact, we would like to test the group with fewer than t(2, n) tests. I will introduce some general constructions of \overline{H} -CFFs and for different types of graphs, such as path, cycle, star, and general rooted trees.

Prangya Parida University of Ottawa ppari017@uottawa.ca

Lucia Moura School of Electrical Engineering and Computer Science (EECS) University of Ottawa Imoura@uottawa.ca

$\mathbf{CP2}$

Coloring Complete Bipartite Graphs with Steiner Triple Systems of Small Circumference

Given a proper n-edge-coloring of the complete bipartite graph $K_{n,n}$, the edges from any two color classes form a partition of the vertices into cycles. Let l(n) be the smallest integer such that there exists a proper n-edge-coloring of $K_{n,n}$ in which all bicolored cycles have length at most l(n). The Hggkvist problem, well studied since its introduction in 1976, concerns finding an upper bound for l(n). While it has long been conjectured that $l(n) \leq 6$, the best known bound to date, due to Benson and Dukes, is l(n) < 182. In this talk, we explore how to label the edges of $K_{n,n}$ using a Steiner triple system on n points. Moreover, we extend the notion of *circumference* from graph theory to Steiner triple systems and show that systems with small circumference lead to proper *n*-edge-colorings of $K_{n,n}$ with small bicolored cycles. Finally, we present and give evidence for a conjecture that there exists a Steiner triple system on npoints with circumference at most 12 for all admissible n.

Justin Z. Schroeder Dakota State University Madison, SD, USA justin.schroeder@dsu.edu

CP3

The Varchenko Determinant for Complexes of Oriented Matroids

Let \mathcal{A} be a real hyperplane arrangement and $L(\mathcal{A})$ the geometric lattice formed by the intersections of hyperplanes in \mathcal{A} . We call the full-dimensional cells of \mathcal{A} topes. The Varchenko Matrix is defined by $\mathcal{V}_{ij} = \prod_{e \in S(T_i, T_j)} w_e$, where w_e are weights on the hyperplanes H_e of the arrangement and $S(T_i, T_j)$ is the set of hyperplanes that have to be crossed on a shortest path from a tope T_i to a tope T_j . Varchenko gave an elegant factorization of the determinant of that matrix, considering the weights as variables:

$$\det(\mathcal{V}) = \prod_{F \in L(\mathcal{A})} (1 - w_F^2)^{m_F}, \qquad (1)$$

where $w_F = \prod_{F \subset H_e} w_e$ and m_F are positive integers depending only on the geometric lattice $L(\mathcal{A})$. We generalized this theorem for a recently introduced combinatorial structure, called Complexes of Oriented Matroids (COMs). COMs can be described by only two axioms that capture local symmetry and local convexity and are a generalization of oriented matroids. In this talk, we will present how the Varchenko matrix generalizes to COMs and explain the general idea of the proof for the factorization formula of its determinant.

Sophia Keip FernUniversität in Hagen sophia.keip@fernuni-hagen.de

Winfried Hochstättler Fakultät für Mathematik und Informatik FernUniversität in Hagen winfried.hochstaettler@fernuni-hagen.de

Kolja Knauer Departament de Matemàtiques i Informàtica Universitat de Barcelona kolja.knauer@ub.edu

CP3

Graph Systoles, Regular Matroids, and Applications to Geometry

Recent work of Michael Wiemeler, Burkhard Wilking, and the presenter has found a link between combinatorial objects called regular matroids and algebraic objects called torus representations that have the geometric property that all isotropy groups are connected. By analyzing the weights of the torus representation as a point cloud in Euclidean space and noting that the matrix whose columns are these vectors is totally unimodular, we can apply Seymours classification to classify these representations. Furthermore, we use this classification to estimate geometric invariants of the torus representation in terms of combinatorial invariants of the corresponding regular matroid. The latter can then be reduced to computations of graph theoretic invariants, which we compute for small graphs and in the process obtain new results in this setting for graphs with small Betti number. The applications to geometry are significant. A highlight the analysis of these representations has been the first proof of Hopfs Euler characteristic positivity conjecture from the 1930s for metrics invariant under a torus action where the torus rank is independent of the manifold dimension.

Lee Kennard Syracuse University ltkennar@syr.edu

$\mathbf{CP3}$

Baker-Bowler Theory for Lagrangian Grassmannians

Baker and Bowler (Matroids over partial hyperstructures, 2019) showed that the Grassmannian can be defined over tracts, a ring-like structure generalizing both partial fields and hyperfields. This notion unifies theories for matroids over partial fields, valuated matroids, and oriented matroids. We extend Baker-Bowler theory for the Lagrangian Grassmannian which is the set of maximal isotropic subspaces of a 2n-dimensional symplectic vector space. By Boege et al. (The geometry of gaussoids, 2019), the Lagrangian Grassmannian is parameterized into the projective space of dimension $2^{n-2}(4 + \binom{n}{2}) - 1$ and its image is exactly the solutions of quadrics induced by determinantal identities of principal and almost-principal minors of a symmetric matrix. From the idea that the strong basis exchange axiom of matroids captures the combinatorial essence of the Plücker relations, we define matroid-like objects, called antisymmetric matroids, from the quadrics for the Lagrangian Grassmannian. We provide its cryptomorphic definition in terms of circuits and show that the Lagrangian Grassmannian can be defined over tracts. Our proof involves a homotopy theorem for graphs associated with antisymmetric matroids, generalizing Maurer's homotopy theorem for matroids. We also prove that if a point in the projective space satisfies 3-/4-term quadrics and its supports form the bases of an antisymmetric matroid, then

it satisfies all quadrics.

Donggyu Kim KAIST University donggyu@kaist.ac.kr

$\mathbf{CP3}$

Cyclic Orderings of Paving Matroids

A matroid of rank r is cyclically orderable if there is a cyclic permutation of its elements such that any r consecutive elements form a basis. It was conjectured by Kajitani, Miyano and Ueno that a matroid M is cyclically orderable iff for all subsets X of positive rank, $\frac{|X|}{r(X)} \leq \frac{|E(M)|}{r(M)}$. In this talk, we describe how one can prove this conjecture for paving matroids.

Sean Mcguinness Thompson Rivers University smcguinness@tru.ca

CP4

On Characterization of Laplacian Spectrum with Well-Known Indices: A Novel Correlation for Hypergraphs

In mathematical chemistry and graph theory, a topological index is a numerical descriptor that measures a molecule's structural characteristics without considering its threedimensional layout. One of the most important things to consider while investigating topological indices is their ability to discriminate between various structures. In view of this, first, we aim to broaden the characterization of extremal graphs, achieving sharp bounds for the normalized Laplacian in terms of Randić index in the existing literature. Second, we suggest some open problems contributing to extremal graph theory to extend our results. Third, we enunciate correlations between our established results and the well-known First Zagreb index in a novel way. Finally, for the first time in the existing state of the art, we explore how to apply Rayleigh's quotient principle to construct a relationship between normalized Laplacian energy and well-known indices for k- uniform hypergraphs, offering a fresh perspective on extremal characterization.

Farwa Asmat Peking University Peking University Farwa_asmat@outlook.com

$\mathbf{CP4}$

A Nordhaus-Gaddum Type Problem for the Normalized Laplacian Spectrum and Graph Cheeger Constant

Nordhaus-Gaddum type problems address questions involving a graph parameter for a graph and its complement. We will look at Nordhaus-Gaddum type problems relating to the second eigenvalue of the normalized Laplacian matrix of a graph. Such questions quantify how poorly connected both a graph and its complement can be. We will present some recent results and conjectures related to these questions.

Mark Kempton Brigham Young University mkempton@math.byu.edu

$\mathbf{CP4}$

On the Sum of the Laplacian Eigenvalues of Graphs

Let G(V, E) be a simple graph of order n, size m and having the vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G) = \{e_1, e_2, \ldots, e_m\}$. The adjacency matrix $A = (a_{ij})$ of G is a (0, 1)-square matrix of order n whose (i, j)-entry is equal to 1 if v_i is adjacent to v_j and equal to 0, otherwise. Let $D(G) = diag(d_1, d_2, \ldots, d_n)$ be the diagonal matrix associated to G, where $d_i = \deg(v_i)$, for all $i = 1, 2, \ldots, n$. The matrix L(G) = D(G) - A(G) is called the Laplacian matrix and its eigenvalues are called the Laplacian eigenvalues of the graph G. We discuss about the Laplacian eigenvalues of certain family of graphs. Let $0 = \mu_n \leq \mu_{n-1} \leq \cdots \leq \mu_1$ be the Laplacian eigenvalues of G and $S_k(G) = \sum_{i=1}^k \mu_i, \ k = 1, 2, \ldots, n$ be the sum of k largest Laplacian eigenvalues of G. For any $k, \ k = 1, 2, \ldots, n$, A. Brouwer conjectured that $S_k(G) = \sum_{i=1}^k \mu_i \leq m + {k+1 \choose 2}$. We discuss the Laplacian eigenvalues of graphs, bounds for $S_k(G)$ and the recent developments of the Brouwer's

<u>Shariefuddin Pirzada</u> University of Kashmir pirzadasd@kashmiruniversity.ac.in

 $\mathbf{CP5}$

conjecture.

Hamiltonicity and Related Properties in K_{r+1} -Free Graphs

In this talk, we discuss a new result on best-possible edge density conditions sufficient to imply traceability, Hamiltonicity, chorded pancyclicity, Hamiltonianconnectedness, k-path Hamiltonicity, k-Hamiltonicity, k-Hamiltonian-connectedness, and k-connectedness in K_{r+1} free graphs. The problem of determining the extremal number ex(n, F), the maximum number of edges in an *n*vertex, F-free graph, has been studied extensively since Turan's theorem. Edge density conditions implying these properties also had been found. We bring together these two themes. Equivalently, we introduce variants of the extremal number ex(n, F) in which we require that the graphs not have some Hamiltonian-like property, and we determine their values for $F = K_{r+1}$. We then extend these results to clique density conditions. This talk is based on joint work with Rachel Kirsch.

Aleyah Dawkins George Mason University adawkin@gmu.edu

Rachel Kirsch George Mason University Fairfax, VA rkirsch4@gmu.edu

CP5

Average Sizes of Maximal Matchings

A matching of a graph G is called a maximal matching if it is not properly contained in any matching of G. The average size of a maximal matching of G, $\operatorname{avm}(G)$, is the average value taken over the sizes of all maximal matchings of G. In this talk, we will discuss the problem of determining graphs which have the maximum or minimum value of $\operatorname{avm}(G)$ when G is restricted to particular families of graphs.

Aysel Erey Utah State University aysel.erey@gmail.com

John Engbers Marquette University john.engbers@marquette.edu

$\mathbf{CP5}$

Approximate Cycle Double Cover

The cycle double cover conjecture states that for every bridgeless graph G, there exists a family \mathcal{F} of cycles such that each edge of the graph is contained in exactly two members of \mathcal{F} . Given an embedding of a graph G, an edge e is called a "bad edge" if it is visited twice by the boundary of one face. CDC conjecture is equivalent to bridgeless cubic graphs having an embedding with no bad edge. In this talk, we introduce non-trivial upper bounds on the minimum number of bad edges in an embedding of a cubic graph. We also introduce better upper bounds on the minimum number of bad edges in an embedding of cyclically 2k-edge-connected and cyclically k-edge-connected cubic graphs. Finally, we introduce another approach to the CDC conjecture. Every embedding allows us to make a dual graph. The embedding gives a CDC if the dual graph has no loop. We are studying how to modify the embedding of a cubic graph by doing controlled modifications of the dual graph. The goal of that is to find a dual with a small number, or ideally no loop edge.

<u>Babak Ghanbari</u> Charles University babak@iuuk.mff.cuni.cz

Robert Šámal IÚUK MFF UK – CSI of Charles University in Prague samal@iuuk.mff.cuni.cz

CP5

Bounded Degree Hypergraphs with No Full Rainbow Structures

For a multi-hypergraph G and a partition P of its vertex set (or edge set), there are various results stating that if the size of each class in P is sufficiently large compared to the maximum degree Δ of G, then there exists an independent set (matching) of G that is full rainbow, i.e., that is a transversal of P. For example, Haxell (2001) proved that there exists an independent transversal in a graph Gif each vertex class in P has size at least 2Δ . Aharoni, Berger, and Meshulam (2005) proved that there exists a full rainbow matching in an r-uniform multi-hypergraph Gif each edge class in P has size at least $r\Delta$. Generalizing previous constructions by Szab and Tardos (2006) and others, we describe a general iterative method for producing multi-hypergraphs with no full rainbow structures which are extremal for these theorems. We also explain how this construction method can be used to produce counterexamples to questions about list (edge) coloring based on color degree. This is based on joint work with Penny Haxell.

<u>Ronen Wdowinski</u> University of Waterloo ronen.wdowinski@uwaterloo.ca

 $\mathbf{CP6}$

Maker-Breaker Rado Games for Equations with Radicals

We study two-player positional games where Maker and Breaker take turns to select a previously unoccupied number in $\{1, 2, ..., n\}$. Maker wins if the numbers selected by Maker contain a solution to the equation

$$x_1^{1/\ell} + \dots + x_k^{1/\ell} = y^{1/\ell}$$

where k and ℓ are integers with $k \geq 2$ and $\ell \neq 0$, and Breaker wins if they can stop Maker. Let $f(k,\ell)$ be the smallest positive integer n such that Maker has a winning strategy when x_1, \ldots, x_k are not necessarily distinct, and let $f^*(k,\ell)$ be the smallest positive integer n such that Maker has a winning strategy when x_1, \ldots, x_k are distinct. When $\ell \geq 1$, we prove that, for all $k \geq 2$, $f(k,\ell) = (k+2)^{\ell}$ and $f^*(k,\ell) = (k^2+3)^{\ell}$; when $\ell \leq -1$, we prove that $f(k,\ell) = [k + \Theta(1)]^{-\ell}$ and $f^*(k,\ell) = [\exp(O(k \log k))]^{-\ell}$. Our proofs use elementary combinatorial arguments as well as results from number theory and arithmetic Ramsey theory.

<u>Collier Gaiser</u>, Paul Horn University of Denver collier.gaiser@du.edu, paul.horn@du.edu

CP6

So Long Sucker, the Game: Two-Player Two-Color Case

So Long Sucker is a 1950 strategy board game developed by mathematician John Forbes Nash Jr. and his colleagues. It has a simple layout consisting of 4 players with k chips each of their designated color, and an empty board with rempty rows. With a clear setup comes intricate rules that allow the game to reflect real life negotiations and conflicts between multiple parties. It is a useful tool to study players behaviors in situations that involve individual and group decisions. The set of rules is very distinctive: players take turns but not in a fixed order, agreements can be made and broken at any time, and a player can win the game even if they were out of chips. One of the main points of interest in studying this game, is to study when a player has a winning strategy. The game starts off with four players that get eliminated one after the other until only the winner is left. Thus in order to study winning strategies, it is of interest to look at endgame situations; particularly when there are only two players left in the game. During this talk, we will present a particular setup of the game: there are two players, first player Blue and second player Red, and their respective colors left in play. We will show through inductive reasoning, how we are able to characterize Blues winning strategies.

<u>Marie Rose Jerade</u> University of Ottawa University of Ottawa mjera100@uottawa.ca

Jean-Lou De Carufel University of Ottawa jdecaruf@uottawa.ca

Groups: A Matrix Method

A classic problem in graph theory is to find the chromatic number of a given graph: that is, to find the smallest number of colors needed to assign every vertex a color such that whenever two vertices are adjacent, they receive different colors. This problem has been studied for many families of graphs, including cube-like graphs, unit-distance graphs, circulant graphs, integer distance graphs, Paley graphs, unit-quadrance graphs, etc. All of those examples just listed can be regarded as abelian Cayley graphs, that is, Cayley graphs whose underlying group is abelian. Our goal is to create a unified, systematic approach for dealing with problems of this sort, rather than attacking each individually with ad hoc methods. Building upon the work of Heuberger, we associate an integer matrix to each abelian Cayley graph. In certain cases, such as when the matrix is small enough, we can more or less read the chromatic number directly from the entries of the matrix. In this way we immediately recover both Payans theorem (that cubelike graphs cannot have chromatic number 4) as well as Zhus theorem (which determines the chromatic number of six-valent integer distance graphs). The proofs utilize only elementary group theory, elementary graph theory, elementary number theory, and elementary linear algebra.

<u>Michael Krebs</u> California State University mkrebs@calstatela.edu

$\mathbf{CP7}$

Strong Odd Coloring of Sparse Graphs

An odd coloring of a graph G is a proper coloring of G such that for every non-isolated vertex v, there is a color appearing an odd number of times in $N_G(v)$. Odd coloring of graphs was studied intensively in recent few years. In this paper, we introduce the notion of a strong odd coloring, as not only a strengthened version of odd coloring, but also a relaxation of square coloring. A strong odd coloring of a graph G is a proper coloring of G such that for every non-isolated vertex v, if a color appears in $N_G(v)$, then it appears an odd number of times in $N_G(v)$. We denote by $\chi_{so}(G)$ the smallest integer k such that G admits a strong odd coloring with k colors. We prove that if G is a graph with $mad(G) \leq \frac{20}{7}$, then $\chi_{so}(G) \leq \Delta(G) + 4$, and the bound is tight. We also prove that if G is a graph with $mad(G) \leq \frac{30}{11}$ and $\Delta(G) \geq 4$, then $\chi_{so}(G) \leq \Delta(G) + 3$.

Hyemin Kwon Ajou university khmin1121@ajou.ac.kr

Boram Park Ajou University borampark@ajou.ac.kr

$\mathbf{CP7}$

Unique Group Coloring of Graphs

Group Coloring is a "non-homogeneous" generalization of vertex coloring, where vertices are colored with group elements in such a way that adjacent vertices do not differ by a value assigned to the edge they share. This talk will discuss what it means for a group coloring to be unique, as an extension of the previously well-studied unique vertex coloring problem, and present some properties of graphs that can be uniquely colored under certain assignments.

Lucian C. Mazza Oakland University lmazza@oakland.edu

$\mathbf{CP7}$

Proper Chromatic Number of Graphs with Small Chromatic Number

The proper orientation number $\vec{\chi}(G)$ of a graph G is the minimum k such that there exists an orientation of the edges of G with all vertex-outdegrees at most k and such that for any adjacent vertices, the outdegrees are different. In [1] it has been proven that the proper chromatic number can be bound for every graph by $\frac{1}{2}MAD(G) + \frac{3\chi(G)\log\chi(G)}{\log\log\chi(G)}$. where MAD is the maximum average degree of the graph. The paper also presents upper bounds for bipartite graphs and outerplanar graphs. In the talk, we will present an elementary proof of the result for bipartite graphs and improve the upper bound for outerplanar graphs from 10 to 9. We use a simple greedy procedure to generate the proper orientations and compute them in linear time. The method can be extended to provide better general bounds for graphs with small chromatic number. [1] Yaobin Chen, Bojan Mohar, Hehui Wu, Proper orientations and proper chromatic number, Journal of Combinatorial Theory, Series B, Volume 161, 2023, Pages 63-85, ISSN 0095-8956, https://doi.org/10.1016/j.jctb.2023.02.003.

<u>Marcin Witkowski</u> Adam Mickiewicz University mw@amu.edu.pl

Andrzej Czygrinow Arizona State University andrzej.czygrinow@asu.edu

Michal Hanckowiak Adam Mickiewicz University mhanckow@amu.edu.pl

Felix Schroe Charles University schroder@kam.mff.cuni.cz

Raphael Steiner ETH raphaelmario.steiner@inf.ethz.ch

$\mathbf{CP8}$

Twin-Width of Subdivisions of Multigraphs

For each $d \leq 3$, we construct a finite set F_d of multigraphs such that for each graph H of girth at least 5 obtained from a multigraph G by subdividing each edge at least two times, H has twin-width at most d if and only if G has no minor in F_d . This answers a question of Berg, Bonnet, and Dprs asking for the structure of graphs G such that each long subdivision of G has twin-width 4. As a corollary, we show that the 7×7 grid has twin-width 4, which answers a question of Schidler and Szeider.

Jungho Ahn Korea Institute for Advanced Study junghoahn@kias.re.kr University of Warwick debsoumya.chakraborti@warwick.ac.uk

Kevin Hendrey Institute for Basic Science kevinhendrey@ibs.re.kr

Sang-Il Oum Institute for Basic Science / KAIST sangil@ibs.re.kr

CP8

Reduced Bandwidth: a Qualitative Strengthening of Twin-width in Minor-closed Classes (and Beyond)

In a reduction sequence of a graph, vertices are successively identified until the graph has one vertex. At each step, when identifying u and v, each edge incident to exactly one of u and v is coloured red. Bonnet, Kim, Thomass and Watrigant [J. ACM 2022] defined the twin-width of a graph G to be the minimum integer k such that there is a reduction sequence of G in which every red graph has maximum degree at most k. For any graph parameter f, we define the reduced f of a graph G to be the minimum integer ksuch that there is a reduction sequence of G in which every red graph has f at most k. Our focus is on graph classes with bounded reduced bandwidth, which implies and is stronger than bounded twin-width. We show that every proper minor-closed class has bounded reduced bandwidth, which is qualitatively stronger than an analogous result of Bonnet et al. for bounded twin-width. We separate twinwidth and reduced bandwidth by showing that any infinite class of expanders excluding a fixed complete bipartite subgraph has unbounded reduced bandwidth, while there are bounded-degree expanders with twin-width at most 6.

Édouard Bonnet Univ Lyon, CNRS, ENS de Lyon, France edouard.bonnet@ens-lyon.fr

O-Joung Kwon Hanyang University ojoungkwon@hanyang.ac.kr

David Wood Monash University, Australia david.wood@monash.edu

CP8

A New Width Parameter of Graphs Based on Edge Cuts: α -Edge-Crossing Width

We introduce graph width parameters, called α -edgecrossing width and edge-crossing width. These are defined in terms of the number of edges crossing a bag of a tree-cut decomposition. They are motivated by edge-cut width, recently introduced by Brand et al. (WG 2022). We show that edge-crossing width is equivalent to the known parameter tree-partition-width. On the other hand, α -edgecrossing width is a new parameter; tree-cut width and α edge-crossing width are incomparable, and they both lie between tree-partition-width and edge-cut width. We provide an algorithm that, for a given *n*-vertex graph *G* and integers *k* and α , in time $2^{O((\alpha+k)\log(\alpha+k))}n^2$ either outputs a tree-cut decomposition certifying that the α -edgecrossing width of *G* is at most $2\alpha^2 + 5k$ or confirms that the α -edge-crossing width of *G* is more than *k*. As applications, for every fixed α , we obtain FPT algorithms for the LIST COLORING and PRECOLORING EXTENSION problems parameterized by α -edge-crossing width. They were known to be W[1]-hard parameterized by tree-partitionwidth, and FPT parameterized by edge-cut width, and we close the complexity gap between these two parameters.

Myounghwan Lee, Yeonsu Chang, O-Joung Kwon Hanyang University

sycuel@hanyang.ac.kr, yeonsu@hanyang.ac.kr, ojoungkwon@hanyang.ac.kr

$\mathbf{CP9}$

Entropy-Variance Inequalities for Discrete Log-Concave Random Variables via Degree of Freedom

Motivated by a localization result of Fradelizi-Gudon (2004), we adapt the notion of degree of freedom to study the extreme points of the set of discrete log-concave probabilities satisfying multiple linear constraints. We find that the extreme points are necessarily piecewise log-affine. As an application, we prove a sharp min-entropy-variance inequality for integer-valued log-concave random variables. More specifically, we show that the geometric distribution minimizes the min-entropy within the class of log-concave probability sequences with fixed variance. This implies a discrete Rnyi entropy power inequality in the log-concave setting, which improves and extends a result of Bobkov-Marsiglietti-Melbourne (2022).

<u>Heshan Aravinda</u> Willamette University haravinda@willamette.edu

CP9

Fixed-Point-Free Involution Factorizations

The problem of factoring random permutations into the composition of two involutions with a prescribed number of fixed points has connections to arithmetic dynamics, comparative genomics, and the analysis of perfect shuffle algorithms. Given a permutation σ in the symmetric group S_n , let invol(σ) denote the number of ways that σ can be written as the product of two involutions in S_n . In an earlier joint paper with Eric Schmutz, we proved that invol is asymptotically lognormal for uniform random permutations. In this talk though, we will consider the permutations that admit fixed-point-free involution factorizations, which are precisely the permutations with an even number of k-cycles for k = 1, 2, ... Through a combination of singularity analysis, the method of moments, and an appeal to the classical Shepp-Lloyd model, we will see that the conditional distribution of the number of fixed-pointfree involution factorizations over such permutations has a discrete limit law instead of a log-Gaussian one.

<u>Charles D. Burnette</u> Department of Mathematics Xavier University of Louisiana cburnet2@xula.edu

CP9

Joint Convergence of Monochromatic Subgraphs in Randomly Colored Dense Multiplex Networks

Given a sequence of graphs $\{G_n\}_{n\geq 1}$ and fixed graph H, denote by $T(H, G_n)$ the number of monochromatic copies of the graph H in G_n in a uniformly random c_n -coloring of the vertices of G_n . In this paper we study the joint 9

distribution of monochromatic subgraphs for dense multiplex networks, that is, networks with multiple layers. Specifically, we consider the joint distribution of $T_n :=$ $(T(H, G_n), T(H', G'_n))$, for two sequences of dense graphs $\{G_n\}_{n\geq 1}$ and $\{G'_n\}_{n\geq 1}$ on the same set of vertices and two fixed graphs H and H'. Under a new notion of joint convergence of the graphs G_n and G'_n in the cut metric, we show that when the number of $c_n = c$ is fixed, the limiting distribution of T_n is the sum of two independent components, one of which is a bivariate Gaussian and the other is a sum of bivariate stochastic integrals. On the other hand, when the number of colors $c_n \to \infty$ (such that $\mathbb{E}[T_n] \to \infty$), then the asymptotic distribution of T_n is a bivariate normal. This generalizes the classical birthday problem, which involves understanding the asymptotics of $T(K_s, K_n)$, the number of monochromatic s-cliques in a complete graph K_n (s-matching birthdays among a group of n friends), to general monochromatic subgraphs in multiplex networks.

<u>Mauricio Daros Andrade</u>, Bhaswar Bhattacharya University of Pennsylvania Department of Statistics and Data Science daros@wharton.upenn.edu, bhaswar@wharton.upenn.edu

CP9

Shuffle Sorting with a Cut after the Longest Increasing Prefix

Cutting a permutation and shuffling the two resulting subpermutations has been studied under a variety of circumstances and applications. We introduce four deterministic algorithms PRE, MIN, PRE-REV, and MIN-REV that each place the cut immediately following the longest increasing prefix of the permutation. Our algorithms consider two natural ways to prioritize how the permutation is then shuffled back together with either the second subpermutation in its original order or its reversal. These algorithms can also be described in terms of systems of queues and stacks. For each algorithm, we characterize and enumerate the sets of permutations that are sorted after k iterations of the algorithm. For three of the four algorithms, the permutations which are sortable after at most k shuffles form permutation classes, that is they can be characterized in terms of classical pattern avoidance. Additionally, the generating function for number of permutations sorted after k iterations of PRE-REV can be shown to be rational because its insertion encoding is a regular language. All four algorithms are shown to be sorting algorithms in the sense that all permutations will be sorted by some finite number of iterations of each algorithm. We can also provide the worst such case for each algorithm for permutations in S_n for any n. Some of our enumerative results include connections to the Eulerian numbers and Stirling numbers of the second kind.

<u>Rebecca Smith</u> SUNY Brockport Brockport, NY, USA rnsmith@brockport.edu

Lara Pudwell Valparaiso University Valparaiso, IN, USA lara.pudwell@valpo.edu

CP10

On the On-line Coloring of Proper Interval Graphs

We consider the on-line coloring problem restricted to proper interval graphs with known interval representation. Chrobak and Slusarek (1981) showed that the greedy First-Fit algorithm has a strict competitive ratio of 2. It remains open whether there is an on-line algorithm that performs better than First-Fit. Piotr (2008) showed that if the representation is not known, there is no better on-line algorithm. Epstein and Levy (2005) showed that no on-line algorithm has a strict competitive ratio less than 1.5 when a unit-interval or proper interval representation is known, which was later improved to 1.6 by Piotr (2008) [for proper intervals] and Biro and Curbelo (2023) [for unit-intervals]. In this paper, we show that there is no on-line algorithm for coloring proper interval graphs with known interval representation with strict competitive ratio less than 1.75 by presenting a strategy that can force any on-line algorithm to use 7 colors on a proper interval graph G with chromatic number $\chi(G) \leq 4$ with known interval representation.

Israel R. Curbelo Kean University Union, NJ icurbelo@kean.edu

Hannah Malko Kean University hmalko@kean.edu

CP10

Approximating the Jump Number of Posets

How close is a given poset to being a linear order? Two metrics suggest themselves: 1. Deconstruct the poset by removing chains the smallest such number of chains is the width of the poset, and small width is close to linear. but likely no linear extension displays the width directly. 2. Deconstruct the poset by shelling chains the smallest such number of chains is the jump number (plus 1). Shelling requires that we cannot remove a chain unless all its lower covers are in the chain itself.; hence the chains in their shelling order form a linear extension. The width of a poset is easily found in polynomial time, whereas the jump number is NP-hard. Jump number has been well studied in scheduling contexts where it is important to avoid set-up costs incurred when unrelated jobs are scheduled consecutively. To date, efficient jump-number algorithms, whether exact or approximate, have been found for only a very few classes of posets. We prove that greedy linear extensions approximately solve jump number in the general case, where the approximation ratio achieved is bound by the smallest value of k for which the poset is $C_2 + C_k$ -free. This represents the first known polynomial time approximation results for general posets, and generalizes a recent result by the author showing that greed achieves approximation ratio 2 when applied to interval posets.

gara.pruesse@viu.ca

CP11

Graph Embeddings and Systole Bounds

While obstructions to embedding graphs into the plane and the real projective plane are well understood, there is no known complete list for other surfaces such as the torus or the Klein bottle. In this talk, we will discuss ongoing work towards a conceptual proof of Chambers' computer assisted results regarding cubic torus obstructions with small first Betti number. Moreover, we will highlight a connection between the existence of embeddings of such graphs into certain surfaces and geometric data of special torus representations shown by Kennard, Wiemeler, and Wilking.

<u>Marie Kramer</u> Syracuse University mkrame04@syr.edu

CP11

On the Oriented Diameter of Near Planar Triangulations

We show that the oriented diameter of any *n*-vertex 2connected near triangulation is at most $\lceil \frac{n}{2} \rceil$ (except for seven small exceptions), and the upper bound is tight. This extends a result of Wang et.al. on the oriented diameter of maximal outerplanar graphs, and improves an upper bound of $n/2 + O(\sqrt{n})$ on the oriented diameter of planar triangulations by Mondal, Parthiban and Rajasingh.

<u>Xiaonan Liu</u> Vanderbilt University xiaonan.liu@vanderbilt.edu

CP11

Hypergraph Supports

For a hypergraph (X,E), a support is a graph Q on X s.t. for all e in E, the induced subgraph Q[e] of Q is connected. We consider hypergraphs defined on a host graph. Given a host graph G = (V,F) with any 2-coloring of V by r,b and a collection of connected subgraphs H of G, a primalsupport is a graph Q on vertices colored b, s.t. for all h in H, induced subgraph Q[b(h)] on vertices of h colored b, is connected. A dual-support is a graph Q^{*} on H s.t. for all v in V, induced subgraph Q^{*}[Hv] is connected, for Hv=h in H:v lies in h. Given two families H and K of connected subgraphs of G, an intersection-support Q is a graph on H s.t. for any k in K, $k_H = \{h \text{ in } H : h \text{ intersects } k\},\$ induces a connected subgraph of Q. We present sufficient conditions on the host graph G and subgraphs so that the resulting primal/dual/intersection support comes from a restricted family in the following two cases: 1) If G has genus g and subgraphs satisfy a topological condition of being cross-free, then there are primal, dual and intersection supports of genus at most g. 2) If G has treewidth log(t) and subgraphs satisfy a combinatorial condition of being non-piercing, then there are primal, dual supports of treewidth O(t). This exponential blow-up is sometimes necessary. We also gave a fast algorithm for points and non-piercing axis parallel rectangles for primal setting. Finally, we show applications to packing, covering and coloring problems on geometric hypergraphs.

Ambar Pal Johns Hopkins University ambar@jhu.edu Rajiv Raman Indraprastha Institute of Information Technology, Delhi rajiv@iiitd.ac.in

Saurabh Ray NYU Abu Dhabi saurabh@nyu.edu

Karamjeet Singh

Indraprastha Institute of Information Technology, Delhi karamjeets@iiitd.ac.in

MS1

The Sampling Lovasz Local Lemma

The Lovasz Local Lemma (LLL), a cornerstone of the probabilistic method, is used to establish the existence of objects, for instance, a satisfying assignment for a k-SAT formula in which no variable appears in too many clauses. In a celebrated work, Moser and Tardos gave a completely algorithmic proof of the LLL, thereby showing (under mild assumptions) that objects whose existence is guaranteed by the LLL can be found in polynomial time. In recent years, the counting/sampling analogue of this question has gained much attention, for instance, for a k-SAT formula satisfying LLL(-like) conditions, can we approximately count the number of satisfying assignments in polynomial time? In this talk, I will provide an introduction to this line of work and discuss recent progress.

<u>Vishesh Jain</u> University of Illinois Chicago visheshj@uic.edu

Huy Tuan Pham, Thuy-Duong Vuong Stanford University huypham@stanford.edu, tdvuong@stanford.edu

MS1

Extremal Problems Around the Classical Hermitian Unital

We will discuss a few extremal problems from different sides of combinatorics, which are related in some way or another with the classical Hermitian unital over $\mathbb{F}_{n^2}^2$.

<u>Cosmin Pohoata</u> Emory University cosmin.pohoata@emory.edu

MS1 Lower Bounds for Incidences

We discuss some new bounds for incidences between points and tubes in the plane (and perhaps higher dimensions) with applications to Heilbronn triangle-type problems. Joint with Alex Cohen and Cosmin Pohoata.

Alex Cohen MIT alexcoh@mit.edu.

Cosmin Pohoata Emory University cosmin.pohoata@emory.edu

Dmitrii Zakharov

MIT zakhdm@mit.edu

$\mathbf{MS2}$

Alternative Discrete Models in Signal Processing

Signal Processing and information theory are usually carried out in the context of the Hamming distance and Shannons theory of communication. A number of problems of current interest fall out of the scope of these areas. Recently, several problems in areas such as biology (e.g., species identification based on DNA alone [New Genomic Information Systems (GenISs)], bioinformatics (e.g., prediction of phenotypic features based on) and self-assembly (e.g., self-controlled and self-replicating circuit models of biological growth) have given rise to new models to address them. They include novel models of the deep geometry of DNA spaces [Dimensionality Reduction in Data Science, Chap 7; Human Mol Gen 31:4], of families of self-assembling resistive circuits modeling biological information processing for self-controlled and self-reproducing growth [Scientific Reports 12, 13371], of morphological feature prediction in black fly from genomic data alone [IEEE/ACM Trans. on Comp. Biol. and Bioinf 18:2], and of biological pathogenicity [Mol Genet Genomics 296]. The goal of this mini-symposium is to present a review of representative examples of such problem and solutions to foster cross-disciplinary interactions to simulate further research in these areas. Of particular interest are results on signal processing in graphs, a topic of increasing interest [Discrete Signal Processing on Graphs: Frequency Analysis (2014); IEEE Trans on Signal Proc. 69].

<u>Max Garzon</u> University of Memphis mgarzon@memphis.edu

Yan Yan Wenzhou-Kean University Kean University at Wenzhou yyan@kean.edu

Guanchao Tong Wenzhou-Kean U tguancha@kean.edu

Jiaju Miao Capital One miaojiaju@gmail.com

Fredy Alexander Colorado National U of Colombia facoloradog@unal.edu.co

MS2

Online Ensemble of Financial Models for Optimal Predictive Returns

Asset-specific factors are commonly used to forecast financial returns and quantify asset-specific risk premia. Using various machine learning models, we demonstrate that the information contained in these factors leads to even larger economic gains in terms of forecasts of sector returns and sector-specific risk metrics. To capitalize on the strong predictive ability of individual models for performance in different sectors, we develop a novel online ensemble algorithm that learns to optimize predictive performance. The algorithm continuously adapts over time to determine the optimal combination of individual models by solely analyzing their most recent predictions. This makes it particularly suited for time series problems, rolling window backtesting procedures, and systems of potentially blackbox models. We derive the optimal gain function, express the corresponding regret bounds in terms of the R-squared measure, and derive an optimal learning rate for the algorithm.

<u>Jiaju Miao</u> Capital One miaojiaju@gmail.com

 $\mathbf{MS2}$

Prediction of Regional Sea Level Using Satellite and Tide Gauge Data by Classic Statisticand Deep Learning Algorithms

In this work, we present new data calibration methods for? forecasting systems of regional sea level prediction based on tide gauge data and climate variables using both classical time series models and modern machine learning methods. Although tide gauge data is considered the gold standard in regional sea level measurement, many developing countries do not have enough tide gauge stations along their coastlines. Satellite altimetry data have been available since the 1990s and can provide a full coverage of the regional sea level measurements for the worlds oceans, including all the coastlines. However, the satellite data must be properly calibrated beforehand to be trustworthy. We describe novel calibration methods of satellite data based on known tide gauge data and use said data to forecast regional sea levels. We also make comparisons on regional sea level predictions based on these two data types as well.

Guanchao Tong Wenzhou-Kean U tguancha@kean.edu

MS2

Linear Structure of DNA Spaces and Its Applications

A DNA-based computation can be regarded as a signal transmission from a source (input molecules encoding for the input or message) to a destination (the resulting molecules encoding for the result of the computation) via a channel given by a solvent solution in a test tube that affect the changes of the input molecules and decodes them into the results, sometimes with errors. The transformations are based on Watson-Crick hybridization based on Gibbs energies for duplex formation for which the Hamming distance is not a suitable model. Unlike the traditional error-detecting and correcting codes in classical information theory, it is necessary to develop strategies to generate error-preventing codes to guarantee the correctness of the DNA computation. A coding theory has been developed based on an appropriate metric (the h-distance) of hybridization affinity to develop encoding solutions that minimize or even prevent errors. In this work, we begin an exploration of the geometry of the hybridization landscapes in these DNA spaces of short oligonucleotides (up to length 60.) As an application, we show that the geometry structure of these spaces can be used to produce a family of error-preventing codes of fairly good quality. These codes have proven to have a number of applications in genomics and bioinformatics, e.g. for elucidating the deep structure of DNA oligonucleotides and deciding the pathogenicity of

biological organisms to humans.

<u>Yan Yan</u> Wenzhou-Kean University Kean University at Wenzhou yyan@kean.edu

Max Garzon University of Memphis mgarzon@memphis.edu

MS3

The Turn Density for Daisies and Hypercubes

The Turn density of an*r*-uniform hypergraph H, denoted by $\pi(H)$, is the limit of the maximum density of an*n*-vertex*r*-uniform hypergraph not containing a copy of H, as n tends to infinity. An*r*-daisy is an*r*-uniform hypergraph consisting of the six*r*-sets formed by taking the union of an (r-2)-set with each of the 2-sets of a disjoint 4-set. Bollobs, Leader and Malvenuto, and also Bukh, conjectured that the Turn density of the*r*-daisy is zero. A folklore Turn-type conjecture for hypercubes states that for fixed d the smallest set of vertices of the*n*-dimensional hypercube Q_n that meets every copy of Q_d has density 1/(d+1) as n goes to infinity. In this talk, we show that the Turn density for daisies is positive, and, by adapting our construction, we also disprove the hypercube conjecture mentioned above. Joint work with David Ellis and Imre Leader.

David Ellis University of Bristol david.ellis@bristol.ac.uk

<u>Maria-Romina Ivan</u> University of Cambridge mri25@cam.ac.uk

Imre Leader University of Cambridge United Kingdom leader@dpmms.cam.ac.uk

MS3

Optimal Bounds on the Polynomial Schurs Theorem

Liu, Pach and Sndor recently characterized all polynomials p(z) such that the equation x + y = p(z) is 2-Ramsey, that is, any 2-coloring of \mathbb{N} contains infinitely many monochromatic solutions for x + y = p(z). In this paper, we find asymptotically tight bounds for the following two quantitative questions.

- For $n \in \mathbb{N}$, what is the longest interval [n, f(n)] of natural numbers which admits a 2-coloring with no monochromatic solutions of x + y = p(z)?
- For n ∈ N and a 2-coloring of the first n integers [n], what is the smallest possible number g(n) of monochromatic solutions of x + y = p(z)?

Our theorems determine f(n) up to a multiplicative constant 2 + o(1), and determine the asymptotics for g(n). This is joint work with Hong Liu and Pter Pl Pach

<u>Jaehoon Kim</u> KAIST jaehoon.kim@kaist.ac.kr

Hong Liu

IBS Korea hongliu@ibs.re.kr

Peter Pal Pach Budapest University of Technology and Economics ppp@cs.bme.hu

$\mathbf{MS3}$

Triangle Ramsey Numbers of Complete Graphs

A graph is *H*-Ramsey if every two-coloring of its edges contains a monochromatic copy of *H*. Define the *F*-Ramsey number of *H*, denoted by $r_F(H)$, to be the minimum number of copies of *F* in a graph which is *H*-Ramsey. This generalizes the Ramsey number and size Ramsey number of a graph. Addressing a question of Spiro, we prove that

$$r_{K_3}(K_t) = \begin{pmatrix} r(K_t) \\ 3 \end{pmatrix}$$

for all sufficiently large t. We do so through a result on graph coloring: there exists an absolute constant K such that every r-chromatic graph where every edge is contained in at least K triangles must contain at least $\binom{r}{3}$ triangles in total.

Jacob Fox, <u>Jonathan Tidor</u>, Shengtong Zhang Stanford University jacobfox@stanford.edu, jtidor@stanford.edu, stzh1555

MS3

Rainbow Hamiltonian Cycles in Random Graphs

We show that for every $\gamma > 0$ if $p \gg \frac{\log n}{n^{1-\gamma}}$ then a.a.s. any o(pn)-bounded edge-coloured G(n, p) satisfies the following. If H is an n-vertex subgraph of G(n, p) with minimum degree at least p/2 + o(n) then G has a rainbow Hamiltonian cycle. Our method largely relies on various probabilistic tools and employs absorption ideas and regularity. This is joint work with Peter Allen and Julia Boettcher.

Peter Allen, Julia Boettcher London School of Economics p.d.allen@lse.ac.uk, j.boettcher@lse.ac.uk

Liana Yepremyan Emory University liana.yepremyan@emory.edu

$\mathbf{MS4}$

Overlapping and Robust Edge-Colored Clustering in Hypergraphs

A recent trend in data mining has explored (hyper)graph clustering algorithms for data with categorical relationship types. Such algorithms have applications in the analysis of social, co-authorship, and protein interaction networks, to name a few. Many such applications naturally have some overlap between clusters, a nuance which is missing from current combinatorial models. In this talk we will define Edge-Colored Clustering, a popular and wellstudied model for categorical clustering in hypergraphs, and present three generalizations, each of which allows for a budgeted amount of overlap between clusters. These new models are NP-hard, but admit greedy approximations, as well as bicriteria approximation algorithms, where the second approximation factor is on the budget for cluster overlap. Interestingly, our algorithms include a tuning parameter which allows for a trade-off between the two approximation factors. That is, one may obtain a tighter approximation guarantee on the problem objective at the expense of a looser guarantee on the overlap budget, or vice versa. We will also present parameterized algorithms and hardness results, as well as experimental results from the application of our models to real-world datasets.

<u>Alex Crane</u>, Brian Lavallee University of Utah alex.crane@utah.edu, brian.lavallee@utah.edu

Blair D. Sullivan University of Utah, U.S. sullivan@cs.utah.edu

Nate Veldt Texas A & M University nveldt@tamu.edu

$\mathbf{MS4}$

Densest Subhypergraph: Negative Supermodular Functions and Strongly Localized Methods

Dense subgraph discovery is a fundamental primitive in graph and hypergraph analysis which among other applications has been used for real-time story detection on social media and improving access to data stores of social networking systems. We present several contributions for localized densest subgraph discovery, which seeks dense subgraphs located nearby a given seed sets of nodes. We first introduce a generalization of a recent anchored densest subgraph problem, extending this previous objective to hypergraphs and also adding a tunable locality parameter that controls the extent to which the output set overlaps with seed nodes. Our primary technical contribution is to prove when it is possible to obtain a strongly-local algorithm for solving this problem, meaning that the runtime depends only on the size of the input set. We provide a strongly-local algorithm that applies whenever the locality parameter is at least 1, and show why via counterexample that strongly-local algorithms are impossible below this threshold. Along the way to proving our results for localized densest subgraph discovery, we also provide several advances in solving global dense subgraph discovery objectives. This includes the first strongly polynomial time algorithm for the densest supermodular set problem and a flow-based exact algorithm for a densest subgraph discovery problem in graphs with arbitrary node weights. We demonstrate the utility of our algorithms on several data analysis tasks.

Yufan Huang, David Gleich Purdue University huan1754@purdue.edu, dgleich@purdue.edu

Nate Veldt Texas A & M University nveldt@tamu.edu

MS4

Scalable Edge Clustering of Dynamic Graphs via Weighted Line Graphs

Timestamped relational datasets consisting of records between pairs of entities are ubiquitous in data and network science. For applications like peer-to-peer communication, email, social network interactions, and computer network security, it makes sense to organize these records 14

into groups based on how and when they are occurring. Weighted line graphs offer a natural way to model how records are related in such datasets but for large real-world graph topologies the complexity of building and utilizing the line graph is prohibitive. We present an algorithm to cluster the edges of a dynamic graph via the associated line graph without forming it explicitly.

<u>Michael Ostroski</u> National Security Agency maostro@uwe.nsa.gov

MS4

Algorithms for Edge-Colored Hypergraph Clustering

Given an edge-colored hypergraph, Edge Colored Clustering (ECC) is the (NP-hard) task of assigning nodes to colors so that node and hyperedge colors match as much as possible. Formally, a hyperedge is called "unsatisfied" if it contains a node whose color does not match the hyperedge's color. The goal is to minimize the number of unsatisfied edges. This task is motivated by clustering applications where the class or cluster of an object tends to match the types of multiway interactions it engages in. For example, one may wish to cluster academic researchers into their respective disciplines based on which journals and conferences they tend to publish in—in this setting a co-authored paper at a specific venue is encoded by a hyperedge with a certain color. This talk will discuss a few other applications of the problem and then present several recent approximation algorithms that are "tight" or "optimal" in a few different regards. This includes approximations that (asymptotically) match UGC-hardness lower bounds, linear programming based algorithms that match an integrality gap lower bound, and fast algorithms based on a relationship with the vertex cover problem.

<u>Nate Veldt</u> Texas A & M University nveldt@tamu.edu

MS5

Weak Diameter Choosability of Graphs with An Excluded Minor

Weak diameter coloring is the key notion used in the recent result that determines the asymptotic dimension of minorclosed families of graphs. We consider the list-coloring analog in this talk. For every graph H, we determine the minimum integer k such that every graph that does not contain H as a minor can be colored so that every monochromatic component has bounded weak diameter as long as every vertex has at least k available colors. This result is a common generalization of previous results about weak diameter coloring of graphs with excluded minors, about weak diameter list-coloring of graphs with bounded Euler genus, and about clustered coloring of graphs with bounded maximum degree and with excluded minors.

Joshua Crouch, <u>Chun-Hung Liu</u> Texas A&M University jcrouch@tamu.edu, chliu@tamu.edu

$\mathbf{MS5}$

How Many Faces May We Expect in a Random 2-Cell Embedding of a Graph into Some Surface?

We consider random 2-cell embeddings of graphs in sur-

faces and discuss how many faces are expected by choosing one of the 2-cell embeddings uniformly at random. (This is essentially the same as discussing the average genus.) In particular, this will be discussed for some concrete graphs (e.g. complete graphs) and for some random graph families. Joint work with J. Campion Loth, K. Halasz, T. Masarik, and R. Samal.

Bojan Mohar

Simon Fraser University mohar@sfu.ca

MS5

Structural Characterizations for Excluded Fat Minors

The classical measure of tree-width seeks to find a treedecomposition of a graph such that the largest bag contains as few vertices as possible. Georgakopoulos and Papasoglu have recently given a structural characterization of when a graph does not admit a tree-decomposition into bags each inducing subgraphs of bounded *radius*. The characterization hinges on so-called *fat minors*, a parameterized generalization of graph minors. Georgakopoulos and Papasoglu have conjectured a general structure theorem on graphs excluding a fixed fat minor. In this talk we present a structural characterization of graphs which do not admit a fat K_4 minor, confirming this case of Georgakopoulos and Papasoglu's conjecture.

Sandra Albrechtsen, Raphael W. Jacobs, Paul Knappe University of Hamburg sandra.albrechtsen@uni-hamburg.de, raphael.jacobs@uni-hamburg.de, paul.knappe@uni-hamburg.de

<u>Paul Wollan</u> University of Rome "La Sapienza" paul.wollan@gmail.com

$\mathbf{MS5}$

Linkages and Removable Paths Avoiding Vertices

We say that a graph G is (2, m)-linked if, for any distinct vertices $a_1, \ldots, a_m, b_1, b_2$ in G, there exist vertex disjoint subgraphs A, B of G such that $\{a_1, \ldots, a_m\}$ is contained in A and $\{b_1, b_2\}$ is contained in B. A fundamental result in structural graph theory is the characterization of (2, 2)linked graphs. It appears to be very difficult to characterize (2, m)-linked graphs for $m \ge 3$. We provide a partial characterization of (2, m)-linked graphs by adding an average degree condition. It implies that (2m+2)-connected graphs are (2, m)-linked. Moreover, if G is a (2m + 2)-connected graph and $a_1, \ldots, a_m, b_1, b_2$ are distinct vertices of G, then there is a path P in G between b_1 and b_2 and avoiding $\{a_1, \ldots, a_m\}$ such that G - P is connected, improving a previous connectivity bound of 10m.

Xingxing Yu School of Mathematics Georgia Tech yu@math.gatech.edu

Xiying Du, Yanjia Li Georgia Institute of Technology xdu90@gatech.edu, yli3557@gatech.edu

Shijie Xie Georgia Institute of TEchnology

$\mathbf{MS6}$

Sparse Directed Networks with Ordering of Out-Edges at Every Node

Government and industry data stores may contain billions of entities, each of which has a type, and directed or undirected pairwise associations, each with a time stamp. The store contains historical data, with periodic addition of new associations. Treat an entity as a vertex in a property graph, which orders its neighbors (via association) using some function of the time stamp. Restricting to the K nearest neighbors of each vertex, or possibly the single nearest neighbor of each type, mitigates the effect of high degree vertices, while creating asymmetric similarity measures not expressible through vectorizatization. We shall demonstrate the unsupervised learning method "rank-based linkage' which satisfies functorial requirements on how results change when new associations are added. A special case arises when each entity is a traffic record e at time t from source x to destination y. Work in an undirected line graph, where records e and f are adjacent if the destination of e is the source of f, and e precedes f in time. Record e orders its neighbors by absolute difference between their time stamps and e's own time stamp. In this case unsupervised learning may be used to reveal clusters of "causally related" traffic records.

Richard Darling National Security Agency U.S. rwdarli@radium.ncsc.mil

$\mathbf{MS6}$

Topological Modeling with Large Hypergraphs and Simplicial Complexes: Challenges and Progress in Computation.

Large hypergraphs and simplicial complexes, some containing hundreds of billions of hyperedges or simplices, have become basic tools for modeling and statistics in the field of data science. Some of the most powerful methods to understand these objects are algebraic: by studying the boundary matrix, for example, we can extract rich feature vectors, localize anomalies, identify communities, etc. However, its challenging to harness these algebraic solutions, computationally. Matrices often have too many nonzero coefficients to store in memory. They are indexed by simplices, rather than integers. They have coefficients in abstract fields and require exact numerical accuracy. They have unusual sparsity patterns. This talk will introduce an open-source library to address some of these problems, Open Applied Topology (OAT). OAT is a highperformance linear algebra solver with a user-friendly front end. It allows the user to perform mathematical operations including matrix/vector addition, multiplication, and factorization (R=DV, RU=D, U-match), and to compute persistence diagrams, barcodes, (optimal) (co)cycle representatives, and induced maps. Users can link the library to new types of chain complexes (simplicial, cubical, etc.), and to Python libraries such as SciPy. Time permitting, we will examine broader trends in topological software development across the research landscape (government, private, academic), and opportunities for future growth and cooperation.

Gregory Henselman-Petrusek Pacific Northwest National Laboratory gregory.roek@pnnl.gov

$\mathbf{MS6}$

Using Mapper Graphs to Understand Complex Data

High dimensional data is ubiquitous in science. Examples include raw data like images, acoustic signals, and environmental measurements. Additionally, the weights and activations of deep learning and foundation models (e.g., LLMs) are extremely high dimensional. The curse of dimensionality makes it difficult to analyze the data in the latent high dimensional space. One tool, from topological data analysis, that is designed to provide a low dimensional graph summary of high dimensional data is mapper. In this talk I will describe mapper, its advantages and limitations, and provide an example of using mapper to summarize high dimensional activations from convolutional neural networks.

Emilie Purvine

Pacific Northwest National Laboratory emilie.purvine@pnnl.gov

MS6

Hyperedge Triplets: Models and Applications

Hypergraphs are a powerful tool to capture group relationships invisible in ordinary graphs. One promising avenue for hypergraph-native analysis is motif discovery as motif patterns serve as building blocks for larger group interactions. Recent work has focused on categorizing and counting hypergraph motifs based on the existence of nodes in hyperedge intersection regions. Here, we argue that the relative sizes of hyperedge intersections within motifs contain varied and valuable information. As a hypergraph alternative to a graph triangle, hyperedge triplets can represent communities which (1) are the least similar with one another, (2) have the highest pairwise but not groupwise correlation, and (3) are the most similar with one another. This presentation will formalize this as a combinatorial optimization problem where we want to maximize the size of a specific region compared to the others. Hyperedge triplets open new avenues for research ranging from author book portfolios to maps of business locations without the use of any metadata.

Jason Niu SUNY University at Buffalo jasonniu@buffalo.edu

Ilya Amburg Cornell University ilya.amburg@pnnl.gov

Sinan G. Aksoy Pacific Northwest National Laboratory sinan.aksoy@pnnl.gov

A. Erdem Sariyuce University at Buffalo erdem@buffalo.edu

MS7

Semi-Streaming Algorithm for Weighted K-Disjoint Matching

We develop two single-pass semi-streaming algorithms for

the maximum weight k-disjoint matching (k-DM) problem. Given an integer k, the k-DM problem is to find k pairwise edge-disjoint matchings such that the sum of the weights of the matchings is maximized. For $k \ge 2$, this problem is NP-hard. Our first algorithm is based on the primal-dual framework of a linear programming relaxation of the problem and is $\frac{1}{3+\varepsilon}$ -approximate. We also design a $\frac{k}{(2+\varepsilon)(k+1)}$ approximate semi-streaming algorithm for k-DM using bmatching. We compare our two algorithms to state-of-theart offline algorithms on 82 real-world and synthetic test problems. On the smaller instances, our streaming algorithms used significantly less memory (ranging from $6 \times$ to $114 \times \text{less}$) and were faster in runtime than the offline algorithms. Our solutions were often within 5% of the best weights from the offline algorithms. On a collection of six large graphs with a memory limit of 1 TB, the offline algorithms terminated only on one graph. The best offline algorithm on this instance required 640 GB of memory and 20 minutes to complete. In contrast, our slowest streaming algorithm for this instance took under four minutes and produced a matching that was 18% better in weight, using only 1.4 GB of memory.

<u>S M Ferdous</u> Pacific Northwest National Laboratory sm.ferdous@pnnl.gov

Bhargav Samineni University of Texas at Austin sbharg@utexas.edu

Alex Pothen Purdue University Department of Computer Science apothen@purdue.edu

Mahantesh Halappanavar Pacific Northwest National Laboratory mahantesh.halappanavar@pnnl.gov

Bala Krishnamoorthy Washington State University kbala@wsu.edu

$\mathbf{MS7}$

Batch-Dynamic Algorithms in the Parallel and Concurrent Models

Due to the rapidly changing nature of real-world graphs, it is crucial to design dynamic algorithms that efficiently maintain graph statistics upon updates, since the cost of recomputation from scratch can be too large on graphs with millions or billions of connections. Furthermore, since often many changes happen in a very short span of time, we can improve performance by using parallelism to process batches of updates instead of a single update at a time. This talk presents new graph algorithms in this parallel batch-dynamic setting for a variety of problems including k-core decomposition, subgraph counting, and densest subgraphs. I will also discuss a new concurrent version of the batch-dynamic model where reads are allowed to occur concurrently with synchronous updates.

Quanquan C. Liu Yale University quanquan.liu@yale.edu

$\mathbf{MS7}$

Online List Labeling with Predictions

A growing line of work shows how learned predictions can be used to break through worst-cast barriers to improve the running time of an algorithm. However, incorporating predictions into data structures with strong theoretical guarantees remains underdeveloped. In this talk, I present our work on the online list labeling problem, which shows how the predictions can be leveraged in this fundamental data structure. In the problem, n items arrive over time and must be stored in *sorted order* in an array of size $\Theta(n)$. The array slot of an element is its *label* and the goal is to maintain sorted order while minimizing the total number of elements moved (i.e., relabeled). We design a new list labeling data structure and bound its performance in two models. In the worst-case learning-augmented model, we give guarantees in terms of the error in the predictions. Our data structure provides strong theoretical guarantees-it is optimal for any prediction error and guarantees the bestknown worst-case bound even when the predictions are entirely erroneous. We also consider a stochastic error model and bound the performance in terms of the expectation and variance of the error. Finally, the theoretical results are demonstrated empirically on temporal real datasets, where the predictions are constructed using the elements that arrive early.

Samuel McCauley Williams College sam@cs.williams.edu

Benjamin Moseley, <u>Aidin Niaparast</u> Carnegie Mellon University moseleyb@andrew.cmu.edu, aniapara@andrew.cmu.edu

Shikha Singh Williams College, U.S. shikha@cs.williams.edu

MS7

Secure Peer-to-Peer Matrix Multiplication for 100+ Players

We describe secure multiparty computation (MPC) algorithms for matrix multiplication. Our algorithms are motivated by the challenges of large networks of batterypowered drones: many players, vulnerability to fail-stop faults, and severe bandwidth limitations. Given shares of two matrices distributed among a large number of players n, we describe an algorithm to compute shares of their product. Our algorithm is scalable in n: each player sends and receives amortized O(1) messages per matrix multiplication for $\Omega(\sqrt{n})$ matrix multiplications. The algorithm is private against coalitions of size $\Theta(\sqrt{n})$, and tolerates $\Theta(\sqrt{n})$ fail-stop faults. We implement our matrix multiplication algorithm using Cicada, a general-purpose open-source MPC software library, and demonstrate that it works well in practice as the engine of a privacy-preserving linear regression method with n = 100.

Jonathan Berry Sandia National Laboratories, U.S. jberry@sandia.gov

Anand Ganti, Kenneth Goss, Carolyn Mayer, Uzoma Onunkwo, Cynthia Phillips Sandia National Laboratories aganti@sandia.gov, kgoss@sandia.gov, cdmayer@sandia.gov, uonunkw@sandia.gov, caphill@sandia.gov

Jared Saia Department of Computer Science University of New Mexico saia@cs.unm.edu

Timothy Shead Sandia National Laboratories tshead@sandia.gov

MS8

Geochromatic Number in Terms of Chromatic Number

Abstract: Let \overline{G} and \overline{H} be geometric graphs. A geometric homomorphism $f: \overline{G} \to \overline{H}$ is a homomorphism $f: G \to H$, that preserves pairs of crossing edges. That is, if edges e_1 and e_2 cross in \overline{G} then $f(e_1)$, $f(e_1)$ must cross in \overline{H} . The minimum n so that there is a geometric homomorphism from \overline{G} to a geometric \overline{K}_n is called the geochromatic number of $X(\overline{G})$. In this talk, we will see some new results on properties of a geometric graph which allow us to bound its geochromatic number in terms of the chromatic number of its underlying abstract graph.

<u>Debra L. Boutin</u> Hamilton College dboutin@hamilton.edu

Alice M. Dean Skidmore College adean@skidmore.edu

$\mathbf{MS8}$

Introduction to Colourings and Homomorphisms

Given graphs G and H, a homomorphism of G to H, denoted $G \to H$, is a vertex mapping that preserves adjacency. As a particular example, $G \to K_n$ is a *n*-colouring of the vertices of G; hence, homomorphisms generalize colourings. We can use the homomorphism framework to define a notion of colouring for other graph like objects. In each case, the vertex mapping preserves the fundamental structure of the object, e.g. for mixed graphs the edge colour and orientation is preserved; for signed graphs the balance of cycles is preserved; and for geometric graphs the crossing of edges is preserved. A common theme explored in this minisymposium is the generalization of *chromatic number* in these new settings. This talk will provide an introduction and overview to the talks in the session with examples from the presenter's own work on signed graphs.

<u>Richard Brewster</u> Department of Mathematics and Statistics Thompson Rivers University rbrewster@tru.ca

$\mathbf{MS8}$

Homomorphisms and Colourings of Mixed Graphs

An (m, n)-mixed graph is obtained from a simple graph by orienting a subset of the edges, then assigning each edge one of m colours and each arc one of n colours. The operation of *switching* at a vertex x of a mixed graph G permutes

the edge colours, arc colours and arc directions according to a permutation π in a fixed permutation group Γ which is a subgroup of $S_m \times S_n \times S_2$. An (m, n)-mixed graph G has a Γ -switchable homomorphism to an (m, n)-mixed graph H if there exists sequence of switches that, when applied to the vertices of G, results in an (m, n)-mixed graph that has a homomorphism to H – a mapping of the vertices of ${\cal G}$ to the vertices of H that preserves edges, arcs and colours. A Γ -switchable k-colouring of an (m, n)-mixed graph G is a Γ -switchable homomorphism of G to some (m, n)-mixed graph H on k vertices. We will discuss the complexity of deciding whether a given (m, n)-mixed graph G has a Γ switchable homomorphism to a fixed (m, n)-mixed graph H, and the complexity of deciding whether a given (m, n)mixed graph G has a Γ -switchable k-colouring, where k is a fixed positive integer.

Richard Brewster Department of Mathematics and Statistics Thompson Rivers University rbrewster@tru.ca

Luke Ingalls, Arnott Kidner, <u>Gary MacGillivray</u> University of Victoria ingallsluke@gmail.com, akidner@uvic.ca, gmacgill@uvic.ca

MS8

Balanced Chromatic Number of Signed Graphs

The balanced chromatic number of a signed graph is the minimum number of parts into which its vertices can be partitioned so that none of the parts induces a negative cycle. This extends the notion of the chromatic number of a graph. We will discuss the notion of balanced chromatic number, introduce a signed version of Hadwiger's conjecture, and see how a 2013 result of Kawarabayashi can be generalized to the signed graph setting: if a signed graph has no negative loop and no subdivision of a complete signed graph on t vertices, then it admits a balanced $\frac{79}{2}t^2$ -coloring.

Andrea Jiménez Universidad de Valparaíso andrea.jimenez@uv.cl

Jessica McDonald Auburn University mcdonald@auburn.edu

Reza Naserasr Institut de recherche en informatique fondamentale CNRS reza@irif.fr

Kathryn Nurse Simon Fraser University knurse@sfu.ca

Daniel A. Quiroz Universidad de Valparaíso daniel.quiroz@uv.cl

MS9

Robust Sublinear Expanders

Expander graphs are perhaps one of the most widely useful classes of graphs ever considered. In this talk, we will focus

on a fairly weak notion of expanders called sublinear expanders, first introduced by Komls and Szemerdi around 30 years ago. They have found many remarkable applications ever since. In particular, we will focus on certain robustness conditions one may impose on sublinear expanders and some applications of this idea.

Matija Bucic Princeton University Institute for Advanced Study mb5225@princeton.edu

MS9

The "Second" Kahn-Kalai Conjecture up to Log Factors

I'll describe some recent (work in) progress on the "Second" Kahn-Kalai Conjecture (KKC2), the original conjecture on graph containment in $G_{n,p}$ that motivated what is now the Park-Pham Theorem (PPT). KKC2 says that $p_c(H)$, the threshold for containing a graph H in $G_{n,p}$, satisfies $p_c(H) = O(p_E \log n)$, where p_E is the smallest p such that the expected number of copies of any subgraph of H is at least one. In other words, for this class of problems, the expectation threshold q in PPT can be replaced by the smaller p_E . We show that $q < O(p_E \log^2 n)$ (implying $p_c(H) = O(p_E \log^3 n)$ via PPT). Joint with Jeff Kahn and Jinyoung Park.

Quentin C. Dubroff Rutgers University qcd2@math.rutgers.edu

Jeff Kahn

Department of Mathematics, Rutgers University Piscataway, New Jersey 08854-8019, USA jkahn@math.rutgers.edu

Jinyoung Park New York University jinyoungpark@nyu.edu

$\mathbf{MS9}$

On the Generalized Ramsey-Turn Density of Cliques

We study the generalized Ramsey-Turn function $\operatorname{RT}(n, K_s, K_t, o(n))$, which is the maximum possible number of copies of K_s in an *n*-vertex K_t -free graph with independence number sublinear in *n*. The case when s = 2 was settled by Erds, Ss, Bollobs, Hajnal, and Szemerdi in the 1980s. We combinatorially resolve the general case for all $s \geq 3$, showing that the (asymptotic) extremal graphs for this problem have simple (bounded) structures. In particular, it implies that the extremal structures follow a periodic pattern when t is much larger than s. Our results disprove a conjecture of Balogh, Liu and Sharifzadeh and show that a relaxed version does hold. This is joint work with Jun Gao, Suyun Jiang, and Hong Liu.

Jun Gao, Suyun Jiang, Hong Liu IBS Korea jungao@ibs.re.kr, jiang.suyun@163.com, hongliu@ibs.re.kr

Maya R. Sankar Stanford University mayars@stanford.edu

MS9

Local Limit Theorem for Joint Subgraph Counts

Extending a previous result of the first two authors, we prove a local limit theorem for the joint distribution of subgraph counts in the Erdos-Rnyi random graph G(n, p). This limit can be described as a nonlinear transformation of a multivariate normal distribution, where the components of the multivariate normal correspond to the graph factors of Janson. As an application, we show a number of results concerning the existence and enumeration of proportional graphs and related concepts, answering various questions of Janson in the affirmative.

Ashwin Sah MIT asah@mit.edu

Mehtaab Sawhney Massachusetts Institute of Technology msawhney@mit.edu

Daniel G. Zhu Princeton University zhd@princeton.edu

MS10

Encapsulaton Structure and Dynamics in Hypergraphs

Hypergraphs are important network models for representing interactions occurring between two or more nodes simultaneously. Given that hyperedges may be of arbitrary size, smaller hyperedges may or may not be subsets of larger hyperedges. Here we study the influence of the subset relationship between hyperedges of different sizes, which we call encapsulation, on a spreading process defined on a hypergraph. For two hyperedges e and e' with sizes |e| < |e'|, we say that e' encapsulates e if $e \subset e'$, *i.e.*, if the smaller hyperedge e is a subset of the larger hyperedge e'. The set of encapsulation relationships in a hypergraph can be viewed as the edges of a *line graph*, where the nodes are the hyperedges and a directed edge indicates that the source hyperedge encapsulates the target. Here we show that simple network analyses of this encapsulation DAG provide information about how interactions of different sizes can influence one another. We then study a simple contagion process over the hyperedges where encapsulation structure is vital to spread. Our work builds on advances in the study of dynamical processes on higher-order structures, including the relationship between spreading dynamics on hypergraphs compared with simplicial complexes, where encapsulation relationships are implied.

Timothy LaRock, Renaud Lambiotte University of Oxford larock@maths.ox.ac.uk, naud.lambiotte@maths.ox.ac.uk

MS10

Uncovering Higher-Order Interactions in the Cortex: Applications of Multivariate Information Theory

re-

The functional connectivity network, constructed from

pairwise dependencies between brain regions, is a wellestablished tool for studying the brain. While powerful, the network model is limited by its inability to capture any higher-order structures. In this talk, I will explore how multivariate information theory reveals higher-order dependencies in the human brain. I will present results from applying the O-information [F.E. Rosas et al., Phys. Rev. E., 100(2019), 032305] to resting state fMRI data, showing that subsystems dominated by synergistic interactions are widespread in the cortex. Highly synergistic subsystems typically sit between canonical functional networks, which are themselves redundancy dominated T.F. Varley and M. Pope et al., Commun. Biol., 6(2023),451]. This result is further supported by an application of the patial entropy decomposition, indicating that canonical organizations of the brain typically capture redundant information[T.F. Varley, M. Pope. et al., Proc. Natl. Acad. Sci., 120(2023), e2300999120]. Finally, I will show that redundancy/synergy dominance varies in both space and time throughout an fMRI scan with notable recurrence of sets of brain regions engaging synergistically. As a whole, my talk will argue that higher-order interactions in the brain are an under-explored space that, made accessible with the tools of multivariate information theory, may offer novel insights.

Maria Pope Indiana University Bloomington popeme@iu.edu

Thomas F. Varley Vermont Complex Systems Center, University of Vermont Dept. of Computer Science, University of Vermont tfvarley@uvm.edu

Joshua Faskowitz Dept. Psychological and Brain Sciences, Indiana University joshua.faskowitz@nih.gov

Olaf Sporns Dept. Psychological and Brain Sciences, Indiana University Luddy Sch. of Informatics, Indiana University osporns@iu.edu

MS10

Stability of Synchronization of Hypergraph Dynamical Systems

Hypergraphs serve as models for capturing group interactions of arbitrary size within complex systems. In this work, we introduce a universal framework for analyzing the stability of synchronization patterns in hypergraph dynamical systems. Utilizing a framework rooted in contraction theory, we derive sufficient stability conditions for synchronization concerning spectral properties of the generalized Laplacian matrix of the hypergraph, and isolate selfdynamics of each node. Assumptions are made regarding the isolated self-dynamics of nodes, and higher-order coupling functions. To illustrate our derived analytical results, we apply the framework to the hypergraph Sherman neuronal model with chemical synaptic interactions and paradigmatic chaotic Lorenz systems. Our framework is applicable to a broad class of models for hypergraph dynamical systems. The establishment of analytical stability conditions represents a pivotal advancement in enhancing comprehension and controlling hypergraph dynamical systems.

<u>Sarbendu Rakshit</u> University at Buffalo sarbendu.math@gmail.com

MS10

On a Generalization of Wasserstein Distance and the Beckmann Problem to Connection Graphs

The intersection of connection graphs and discrete optimal transport presents a novel paradigm for understanding complex graphs and node interactions. In this paper, we delve into this unexplored territory by focusing on the Beckmann problem within the context of connection graphs. Our study establishes feasibility conditions for the resulting convex optimization problem on connection graphs. Furthermore, we establish strong duality for the conventional Beckmann problem, and extend our analysis to encompass strong duality and duality correspondence for a quadratically regularized variant. To put our findings into practice, we implement the regularized problem using gradient descent, enabling a practical approach to solving this complex problem. We showcase optimal flows and solutions, providing valuable insights into the real-world implications of our theoretical framework.

Sawyer Robertson, Dhruv Kohli, Gal Mishne UC San Diego s5robert@ucsd.edu, dhkohli@ucsd.edu, gmishne@ucsd.edu

Alexander Cloninger University of California, San Diego, USA acloninger@ucsd.edu

MS11

Precoloring Extension in Planar Eulerian Triangulations

By a well-known result of Fisk, if G is a triangulation of an orientable surface with exactly two vertices u and v of odd degree, then u and v must have the same color in any 4-coloring of G. This indicates that in order to generalize the Four Color Theorem, one first needs to understand the behavior of 4-colorings of (nearly) eulerian triangulations. With this as a motivation, we study the precoloring extension problem in planar eulerian triangulations, obtaining interesting partial results.

Zdenek Dvorak

Computer Science Institute Charles University, Prague rakdver@iuuk.mff.cuni.cz

MS11

Peaceful and Conflict-Free Colouring

A proper conflict-free colouring of a graph is a colouring of the vertices such that any two adjacent vertices receive different colours, and for every non-isolated vertex v, some colour appears exactly once on the neighbourhood of v. Caro, Petruševski and Škrekovski conjectured that every connected graph with maximum degree $\Delta \geq 3$ has a proper conflict-free colouring with at most $\Delta + 1$ colours. This conjecture holds for $\Delta = 3$ and remains open for $\Delta \geq 4$. We discuss some results related to this conjecture including proving that it holds asymptotically; namely, every graph with maximum degree Δ has a proper conflict-free colouring with $(1 + o(1))\Delta$ colours.

<u>Bruce Reed</u> Academia Sinica bruce.al.reed@gmail.com

Chun-Hung Liu Texas A&M University chliu@tamu.edu

MS11

Correspondence Packings of Planar Graphs

Suppose a graph G has list chromatic number k. It is easy to see that if L is a (k + 1)-list assignment for G, then G admits two L-colourings φ_1 and φ_2 where $\varphi_1(v) \neq \varphi_2(v)$ for every $v \in V(G)$. But what if we want still more disjoint L-colourings without making our lists too big? In this talk, I will discuss recent progress towards determining the list packing number of various classes of planar graphs: that is, the smallest number k such that if L is a k-list assignment for an arbitrary graph G in the class under study, then L can be decomposed into k disjoint L-colourings. All results I will discuss also hold in the correspondence colouring framework. Joint work with Daniel Cranston.

Evelyne Smith-Roberge University of Waterloo Ontario, Canada evelyne.smithroberge@gmail.com

Daniel Cranston Virginia Commonwealth University Department of Mathematics and Applied Mathematics dcranston@vcu.edu

MS11

Tight Minimum Degree Conditions for Apexouterplanar Minors and Subdivisions in Graphs and Digraphs

Motivated by Hadwiger's conjecture, we study graphs H for which every graph with minimum degree at least |V(H)| - 1 contains H as a minor. We prove that a large class of apex-outerplanar graphs satisfies this property. Our result gives the first examples of such graphs whose vertex cover numbers are significantly larger than a half of its vertices, and recovers all known such graphs that have arbitrarily large maximum degree. Our proof can be adapted to directed graphs to show that every directed graph with minimum out-degree at least t contains a certain orientation of the wheel and of every apex-tree on t+1 vertices as a subdivision and a butterfly minor respectively. These results provide the optimal upper bound for the chromatic number and dichromatic number of graphs and directed graphs that do not contain the aforementioned graphs or directed graphs as a minor and butterfly minor, respectively. Special cases of our results solve an open problem of Aboulker, Cohen, Havet, Lochet, Moura and Thomassé and strengthen results of Gishboliner, Steiner and Szabó.

Youngho Yoo, Chun-Hung Liu Texas A&M University yyoo@tamu.edu, chliu@tamu.edu

MS13

Finding Robust Clusters on Graphs Through Con-

sensus Clustering

We develop an algorithm that finds consensus of many different clustering solutions of a graph. We formulate the problem as a median set partitioning problem and propose a greedy optimization technique. By using the graph structure efficiently, our algorithm discovers a consensus clustering much faster than alternative approaches. This enhanced computational efficiency is achieved without compromising the quality of the solution. In order to make it applicable for large graphs, we removed sequential dependencies from our algorithm and designed a parallel algorithm. Our parallel algorithm achieves 50x speedup when utilizing 64 processing cores for synthetic benchmark graphs.

Md Taufique Hussain Indiana University mth@indiana.edu

MS13

Mining Dense Subgraphs in Real-World Graphs

The problem of mining dense subgraphs in a given graph has been an important problem both in theory and in practice, with applications across many domains including social network analysis, bioinformatics, chemistry, neuroscience, fraud analysis, etc. There have been many different definitions of dense that have been used in the literature, but the goal across all such works is to list the dense subgraphs in the graph. There have been 2 main approaches used for this problem. One treats the problem as a clustering problem and employs one of the many algorithms and heuristics available for clustering (eg. Louvain). These are surprisingly fast in practice but do not allow vertices to overlap and essentially find a partition of the vertices. In practice, however, vertices are often present in multiple dense subgraphs simultaneously. The second approach defines the dense subgraph as a motif, eg., a clique or a k-plex, and aims to enumerate all maximal such structures in the graph. Most of these enumeration problems are NP-Hard and hence impractical for large graphs. However, this approach allows vertices to overlap and can lead to better-quality dense subgraphs. In this talk we will explore the question: can we get the best of both worlds? We will talk about some recent advances towards answering this question and discuss some related open problems.

<u>Shweta Jain</u> University of Utah shweta.jain@utah.edu

MS13

Structure Detection in Graphs Using Direction-Optimizing Label Propagation Algorithm

Label Propagation is well-known as a machine learning algorithm for classification, but it is also an effective method for discovering communities and connected components in networks. We present a new Direction-Optimizing Label Propagation Algorithm (DOLPA) framework that enhances the performance of the standard Label Propagation Algorithm (LPA), increases its scalability, and extends its versatility and application scope. As a central feature, the DOLPA framework relies on the use of frontiers and alternates between label-push and label-pull operations to attain high performance. It is formulated in such a way that the same basic algorithm can be used for finding communities or connected components in graphs by only changing the objective function used. In this presentation, we discuss the design and implementation of the enhanced algorithm. We also present experimental evaluation of our implementations using the LFR benchmark and real-world networks drawn from various domains. Compared with an implementation of LPA for community detection available in a widely used network analysis software, we achieve as high as five times the F-Score while maintaining similar runtime for graphs with overlapping communities. For connected component decomposition, our algorithm achieves up to 13x speedup over the Shiloach-Vishkin (SV) algorithm, and up to 1.6x speedup over Afforest on an Intel Xeon processor using 40 threads.

Tony Liu Amazon Web Services xtliu@amazon.com

Andrew Lumsdaine University of Washington al75@uw.edu

Assefaw Gebremedhin Washington State University assefaw.gebremedhin@wsu.edu

MS13

The Case for External Graph Sketching

Algorithms in the graph semi-streaming model use O(V polylog(V)) space, and the fact that these algorithms are mostly not used in practice may be because this space requirement is too large for RAM on todays hardware. While eventually RAM may grow large enough to accommodate these algorithms, why wait? We present a shortcut: a technique to transform graph sketching algorithms into algorithms which can be run on today's hardware. We present a number of semi-streaming linear sketch algorithms which only require O(polylog(V)) RAM, along with O(V polylog(V)) disk. These algorithms are simultaneously space-efficient and also I/O efficient, meaning they can be run on today's SSDs (which are large enough to store the sketches, and fast enough if the data accesses are sequential). We present a technique for transforming a large category of graph sketching algorithm (vertex-based sketches) into I/O efficient sketching algorithms. We also present an I/O lower bound for sketching algorithms in the external memory model.

David Tench Rutgers dtench@pm.me

MS14 Erdos-Hajnal, Holes, and Subdivisions

A hereditary class C of graphs has the Erdos-Hajnal property if there exists c > 0 such that every *n*-vertex graph in C has a clique or stable set of size at least n^c . We prove that for every two cycles C, D, the class of graphs that are both C-free and \overline{D} -free (H-free means having no induced copy of H, and \overline{D} denotes the complement of D) has the Erdos-Hajnal property. This extends and unifies several previous results. We also prove that if J is obtained from a complete graph by subdividing every edge exactly twice, then the class of graphs that are both J-free and \overline{J} -free has the property as well. As an application, this verifies the Erdos-Hajnal property of every class of graphs not containing a fixed graph as a pivot-minor, which was conjectured by Kim and Oum and proved earlier by Davies using different methods. These results are in fact special cases of a more general statement which will be discussed in the talk. Along the way, we provide a short proof of a well-known result of Fox and Sudakov that for every graph H, every H-free graph contains a stable set or complete bipartite subgraph of polynomial size. Joint work with Alex Scott and Paul Seymour.

Tung Nguyen Princeton University tunghn@math.princeton.edu

Alex Scott Oxford scott@maths.ox.ac.uk

Paul Seymour Princeton University pds@math.princeton.edu

MS14

Reuniting $\chi\text{-}\mathsf{Boundedness}$ with Polynomial $\chi\text{-}\mathsf{Boundedness}$

A class \mathcal{F} of graphs is χ -bounded if there is a function f such that $\chi(H) \leq f(\omega(H))$ for all induced subgraphs H of a graph in \mathcal{F} . If f can be chosen to be a polynomial, we say that \mathcal{F} is polynomially χ -bounded. Esperet proposed a conjecture that every χ -bounded class of graphs is polynomially χ -bounded. This conjecture has been disproved; it has been shown that there are classes of graphs that are χ -bounded but not polynomially χ -bounded. Nevertheless, inspired by Esperet's conjecture, we introduce Pollyanna classes of graphs. A class \mathcal{C} of graphs is Pollyanna if $\mathcal{C} \cap \mathcal{F}$ is polynomially χ -bounded for every χ -bounded class \mathcal{F} of graphs. We prove that several classes of graphs are Pollyanna and also present some proper classes of graphs that are not Pollyanna.

Maria Chudnovsky Princeton University mchudnov@math.princeton.edu

Linda Cook Institute for Basic Science lindacook@ibs.re.kr

James Davies University of Cambridge jgd37@cam.ac.uk

Sang-Il Oum Institute for Basic Science / KAIST sangil@ibs.re.kr

MS14

Packing Even Directed Circuits Quarter-Integrally

We prove the existence of a function $f \colon \mathbb{N} \to \mathbb{N}$ such that for every integer k every digraph D either contains a collection C of k directed cycles of even length such that no vertex of D belongs to more than four cycles in C, or there exists a set $S \subseteq V(D)$ of size at most f(k) such that D - S has no directed cycle of even length.

Maximilan Gorsky

Technical University Berlin m.gorsky@pm.me

Ken-ichi Kawarabayashi National Institute of Informatics, Japan k_keniti@nii.ac.jp

Stephan Kreutzer Technical University Berlin stephan.kreutzer@tu-berlin.de

<u>Sebastian Wiederrecht</u> Institute for Basic Science, DIMAG sebastian.wiederrecht@gmail.com